Comment on "Shadow model for sub–barrier fusion applied to light systems"

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We demonstrate that the cross sections derived from the "shadow model" for reactions between light nuclei disagree with low–energy laboratory data and exhibit unphysical behavior at energies below those for which data exist. As a consequence, the large thermonuclear reaction rates obtained by Scalia and Figuera [Phys. Rev. **C46**, 2610 (1992)] are wrong.

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In a recent publication [1], Scalia and Figuera argue that the rates of the nuclear reactions important in solar hydrogen burning are substantially larger than those adopted in the Standard Solar Model [2]. This claim is based on a "shadow model" for the energy dependence of the low–energy cross sections. We demonstrate in this comment that this energy dependence is both incorrect and unphysical.

In many astrophysical scenarios (e.g., our sun), charged–particle nuclear reactions proceed at such low energies that a direct experimental determination of the cross section is not possible with existing techniques. Extrapolation of the measured cross sections to stellar energies is thus necessary. To be trustworthy, such extrapolations should not only be tied closely to experimental information, but should also be guided by a strong theoretical foundation.

For non–resonant reactions of charged particles (e.g., those that take place in solar hydrogen burning), tunneling through the Coulomb barrier dominates the energy dependence of the cross section at the low energies of astrophysical interest, giving rise to a very rapid decrease of the cross section $\sigma(E)$ with decreasing center–of–mass energy E. For a reliable extrapolation, this dominant energy dependence is factored out and the cross section is usually expressed in terms of the astrophysical S–factor:

$$S(E) \equiv \sigma(E) \cdot E \cdot \exp\{2\pi\eta(E)\} \; . \tag{1}$$

The Sommerfeld parameter is given by

$$\eta(E) = \frac{Z_1 Z_2 e^2}{\hbar v} \; , \tag{2}$$

where v is the relative velocity in the entrance channel and Z_1, Z_2 are the charge numbers of the colliding nuclei. The form of Eq. (1) embodies the s–wave tunneling through the Coulomb barrier of two point–like nuclei. In the absence of near–threshold resonances, the energy dependence of the S–factor is expected to be weak, reflecting only effects like the strong interaction between the collision partners, their finite sizes, contributions from other partial waves, the final state phase space, etc.

The physical picture behind the definition (1) has been confirmed in numerous measurements of cross sections for reactions between the light nuclei [3]. As a typical example, Fig. 1 shows the astrophysical S–factor for the $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ reaction, which terminates the ppI–chain in solar hydrogen burning. The S–factor data [4] clearly show only a very weak and smooth energy dependence indicating that the s–wave penetrability through the Coulomb barrier correctly describes the low–energy cross section.

These empirical observations are confirmed in a microscopic study [5] of the low–energy $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ reaction in which the effects of nuclear structure, the strong interaction, antisymmetrization, etc. were taken into account. As indicated by the solid curve, the parameter–free calculated energy–dependence of the S–factor accurately describes the data. Thus, one has some confidence that this more elaborate nuclear model is also capable of extrapolating the astrophysical S–factor to the most effective energy under solar conditions ($E_0 \approx 22$ keV). The calculation yields $S(0) \approx 5.3$ MeV $\cdot$ b, in close agreement with the value used in the standard solar model [2]. This same microscopic model simultaneously (without parameter adjustment) reproduces the measured S–factors of the analogue $^3\mathrm{H}(^3\mathrm{H}, 2n)^4\mathrm{He}$ reaction, demonstrating that the conventional Gamow barrier penetration accounts correctly for the physics of low–energy nuclear reactions.

Applying their shadow model for sub–barrier fusion, Scalia and Figuera [1] obtained low–energy cross sections (and consequently reaction rates at solar temperatures) that are significantly higher than the standard values [2]. Rather than being based on the correct physical picture of barrier penetration, the energy dependence of the low–energy cross section in this model is simply assumed (Eqs. (3–5) and (8,9) in Ref. [1]).

We will demonstrate that this assumption is wrong. Having six fit parameters at their disposal, Scalia and Figuera claim [1] to reproduce the energy dependence of the measured cross sections, and support their claim by numerous figures in which cross sections are plotted as functions of E. However, as these figures use a logarithmic scale to plot the rapidly–varying cross sections, it is difficult to judge the success of the shadow model approach in reproducing the data. To do so more easily, we have transformed the fusion cross sections

as calculated from the shadow model Eqs. (3–9) of Ref. [1] (parameters as given in Table I of that reference) into the S-factor defined by (1). As a typical example, we compare the $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ S–factors predicted by Scalia and Figuera with the most modern and precise data [4]. It is obvious from our figure that the shadow model does not reproduce the energy dependence of the measured $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ data. More importantly, there is an unjustified and unphysical increase of the S–factor at very low energies, leading to the large shadow model reaction rates.

We find similar inaccuracies for the other reactions considered in Ref. [1] and, in each case, the model predicts an unphysical, dramatic increase of the S–factor at energies smaller than those for which data are available. We therefore conclude that the shadow model is not useful for extrapolating measured cross sections to astrophysically relevant energies, and that any conclusions drawn from such extrapolations are unjustified.

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REFERENCES

[1] A. Scalia and P. Figuera, Phys. Rev. **C46**, 2610 (1992).

[2] J. Bahcall, Neutrino Astrophysics (Cambridge, 1983).

[3] W. A. Fowler, Nobel lecture, Rev. Mod. Phys. **56**, 149 (1984).

[4] A. Krauss, H. W. Becker, H. P. Trautvetter, and C. Rolfs, Nucl. Phys. **A467**, 273 (1987).

[5] S. Typel, G. Blüge, K. Langanke, and W. A. Fowler, Z. Phys. A **339**, 249 (1991).

FIGURES

FIG. 1. S–factor for the $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ reaction. Points are the experimental data from Ref. [4]. The solid line is the energy dependence predicted by the microscopic model of Ref. [5], while the dashed curve shows the shadow model prediction of Ref. [1]. The arrow indicates E_0, the "most effective energy" for a temperature of 15×10^6 K.